STUDY OF CHARGED-PARTICLE DYNAMICS DURING EJECTION OF A REACTIVE JET UNDER ORBITAL FLIGHT CONDITIONS

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When the working substance in the forechamber of a control motor on an orbiting spacecraft is ignited, charged particles are produced together with neutral molecules and larger combustion product particles. Control motors usually operate in a pulsed mode. As the jet escapes into the highly rarefied atmosphere the gas passes through all motion regimes: from a continuous medium regime to one in which collision processes can be neglected [1-3]. In the collision-free regime, which is the one we will consider further, a force acts on the particles exiting the motor (in a more general case, upon particles emitted from the surface of the apparatus). For a particle with charge q in a magnetic field $\mathbf{B}$ this force will equal $q[\mathbf{v}\times\mathbf{B}]$, where v is the particle velocity relative to the field. This force can cause a return of the particle to the spacecraft surface. Of special interest is the motion of heavy particles, which under real conditions have radii in the range $\sim 10^{-9}$-10^{-7} m and a density of approximately 2 g/cm^2 (so that their mass M proves to be much higher than the mass of molecules in the atmosphere m). A study of the dynamics of such particles is necessary to determine their role in erosion of vehicle parts, and also in formation of signals in equipment used to detect charged particles. Because of their large mass, these charged particles have a long rotation period in the magntic field, and can collide with the vehicle some time after switchoff of the engine. As will be shown below, aerodynamic resistance forces can have a significant effect on the motion of such particles. In the general case the charge of such particles apparently varies with time, for example, because of precipitation of particles with opposite charge from the ionosphere. In the analysis presented below, we will assume for simplicity, that the charge remains constant. The question of the characteristic neutralization time for particle charge will be evaluated at the end of the study. Moreover, in deriving the various relationships it was assumed that the particle velocity in the jet u is usually small in comparison with the velocity of the vehicle v_0.

For further analysis we will introduce a coordinate system (Fig. 1) with z-axis directed along the vector $\mathbf{V}_0$, x-axis directed vertically up from the planet surface, with the x-, y-, and z-axes forming a right-hand orthogonal coordinate system. For the origin of this system we choose that point of the vehicle trajectory at which its center of mass is located at the time of motor switchon, and we will assume that this origin is then fixed with respect to the magnetic field. We denote by β the angle between the vectors $\mathbf{V}_0$ and $\mathbf{B}$, and by η the normal to the plane in which these vectors lie (so that η, z, and $\mathbf{B}$ form a right-hand system), and by ε the angle between the axes η and y. Let t_1 be the time beginning at which the magnetic field B acts on the charged particle jet, with the particle at this time being located at the point (x_s, y_s, z_s) and having velocity components u_x, u_y, u_z relative to the vehicle.

To consider the friction force F acting on the particle because of molecules of the surrounding medium, we will assume the particle to be a sphere of radius a. Then $\mathbf{F} = -\alpha m\pi a^2 n v^2 \mathbf{v}/v$. Here n is the gas concentration, the coefficient α is close to unity [4] for a velocity ratio $S \geqslant 6$ ($S = v\sqrt{m/2kT_\infty}$, where k is Boltzmann's constant and T_∞ is the gas temperature).

The equations of motion of the particle in the magnetic field have their simplest form in the coordinate system ξ, η, z, the ξ z-axis of which lies in the plane ($\mathbf{V}_0$, $\mathbf{B}$) and is orthogonal to the direction of $\mathbf{V}_0$ [5]. In this coordinate system the modulus of the particle velocity v is equal to

$$v = \left(v_\xi^2 + v_\eta^2 + v_z^2\right)^{1/2}.$$

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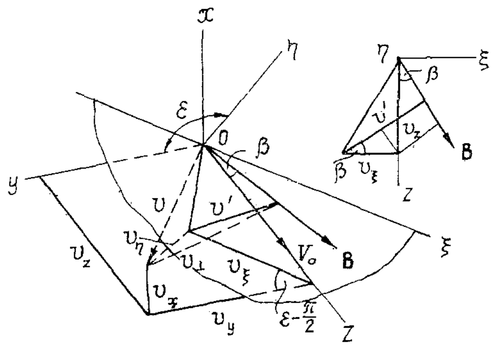

Fig. 1

Under orbital flight conditions the velocity component v_z is close to V_0, while the components v_ξ and v_η are of the order of magnitude of the reactive jet velocity. Therefore $v_z^2 \simeq V_0^2 \gg v_\xi^2 + v_\eta^2$, and consequently, $v \simeq V_0$. Then the components of the friction force along the ξ, η, z axes can be represented in the form

$$F_\xi = -\alpha mn\pi a^2 V_0 v_\xi, \qquad (1)$$
$$F_\eta = -\alpha mn\pi a^2 V_0 v_\eta,$$
$$F_z = -\alpha mn\pi a^2 V_0 v_z.$$

The equations of motion of a particle of mass M under the influence of the magnetic field and aerodynamic resistance forces under the conditions of interest here can be written in the following manner (here and below, a dot above a letter denotes differentiation with respect to time):

$$M\dot{v}_\xi = qv_\eta B \cos\beta + F_\xi; \qquad (2)$$

$$M\dot{v}_\eta = qv_z B \sin\beta - qv_\xi B \cos\beta + F_\eta; \qquad (3)$$

$$M\dot{v}_z = -qv_\eta B \sin\beta + F_z. \qquad (4)$$

Differentiating Eq. (3) with respect to time and its subsequent transformation with consideration of Eqs. (2)-(4) leads to an equation describing damping oscillations

$$\ddot{v}_\eta + 2\delta\dot{v}_\eta + \omega_0^2 v_\eta = 0, \qquad (5)$$

where

$$\delta = \alpha(m/M)n\pi a^2 V_0; \qquad (6)$$

$$\omega_0^2 = \omega^2 + \delta^2, \qquad (7)$$

where $\omega = qB/M$ is the cyclical rotation frequency of the particle in the magnetic field. In the special case considered Eq. (5) does not have an aperiodic solution which corresponds to the condition $\delta^2 \geqslant \omega_0^2$ [6, 7], i.e., $\omega^2 \leqslant 0$, which contradicts the requirement of a real, nonzero value of ω if $B \neq 0$.

The solution of Eq. (5) has the form

$$v_\eta = -v_{\perp 1} e^{-\delta(t-t_1)} \sin\omega'(t-t_0), \qquad (8)$$

where $v_{\perp 1}$ is the projection of the particle velocity on the plane perpendicular to **B**, at time t_1 (in contrast to the case where the resistance force is equal to zero, the quantity $v_\perp$ is a function of time); $\omega' = \sqrt{\omega_0^2 - \delta^2}$; t_0 is the time at which $v_\eta = 0$. Considering Eq. (7), we find that in the case considered $\omega' = \omega$.

Substitution of Eq. (8) in Eqs. (2), (3) and solution of the latter with consideration of initial conditions gives

$$v_\xi = v_{\xi 1} + v_{\perp 1}\Psi \cos\beta; \qquad (9)$$

$$v_z = V_0 + u_z - v_{\perp 1}\Psi \sin\beta. \qquad (10)$$

Here

$$\Psi = e^{-\delta(t-t_1)} \cos\omega(t-t_0) - \cos\omega(t_1-t_0). \qquad (11)$$

Expressions for the velocity components v_x, v_y in the coordinate system xyz can be obtained from the relationships

$$v_\xi = v_x \cos\varepsilon - v_y \sin\varepsilon,\ v_\eta = v_x \sin\varepsilon + v_y \cos\varepsilon, \tag{12}$$

which follow from the geometric constructions shown in Fig. 1. In particular, at time t_1 we have

$$v_{\xi 1} = u_x \cos\varepsilon - u_y \sin\varepsilon. \tag{13}$$

Equating the righthand sides of Eqs. (12), (8), (9) and solving the system of equations obtained simultaneously, we find

$$v_x = u_x \cos^2\varepsilon - u_y(\sin 2\varepsilon)/2 + v_{\perp 1}\Psi \cos\varepsilon \cos\beta - v_{\perp 1}[\Phi + \sin\omega(t - t_1)]\sin\varepsilon; \tag{14}$$

$$v_y = -u_x(\sin 2\varepsilon)/2 + u_y \sin^2\varepsilon - v_{\perp 1}\Psi \sin\varepsilon \cos\beta - v_{\perp 1}[\Phi + \sin\omega(t - t_1)]\cos\varepsilon. \tag{15}$$

Here

$$\Phi = e^{-\delta(t-t_1)} \sin\omega(t - t_0) - \sin\omega(t - t_1). \tag{16}$$

As follows from Fig. 1 (see also, [5]) the projection of the particle velocity $v_{\perp 1}$ is defined by

$$v_{\perp 1}^2 = v_1'^2 + v_{\eta 1}^2 = [(V_0 + u_z)\sin\beta + v_{\xi 1}\cos\beta]^2 + v_{\eta 1}^2. \tag{17}$$

Given the condition that at time t_1 the particle is located at a point with coordinates x_s, y_s, z_s relative to the vehicle, integration of Eqs. (10), (14), (15) gives the following expressions for the current coordinates ($t - t_1 = \tau$):

$$\begin{gathered}[u_x \cos^2\varepsilon - u_y(\sin 2\varepsilon)/2 - v_{\perp 1}\cos\varepsilon\cos\beta \times \\ \times \cos\omega(t_1 - t_0)]\tau + [v_{\perp 1}/(\delta^2 + \omega^2)](\omega\cos\varepsilon\cos\beta + \delta\sin\varepsilon)\Phi + \\ + [v_{\perp 1}/(\delta^2 + \omega^2)](\omega\sin\varepsilon - \delta\cos\varepsilon\cos\beta)\Psi = x - x_s;\end{gathered} \tag{18}$$

$$\begin{gathered}[u_y \sin^2\varepsilon - u_x(\sin 2\varepsilon)/2 + v_{\perp 1}\sin\varepsilon\cos\beta \times \\ \times \cos\omega(t_1 - t_0)]\tau + [v_{\perp 1}/(\delta^2 + \omega^2)](\delta\cos\varepsilon - \omega\sin\varepsilon\cos\beta)\Phi + \\ + [v_{\perp 1}/(\delta^2 + \omega^2)](\omega\cos\varepsilon + \delta\sin\varepsilon\cos\beta)\Psi = y - y_s;\end{gathered} \tag{19}$$

$$[u_z + v_{\perp 1}\sin\beta\cos\omega(t_1 - t_0)]\tau - v_{\perp 1}\sin\beta\frac{\omega}{\delta^2 + \omega^2}\Phi + v_{\perp 1}\sin\beta\frac{\delta}{\delta^2 + \omega^2}\Psi = z - V_0 t - z_s = z_0 - z_s. \tag{20}$$

The expressions obtained can be considered as a system of equations, simultaneous solution of which will permit establishment of the relationship between the time interval τ, the starting coordinates x_s, y_s, z_s, and the finish coordinates x, y, z_0, as well as the initial particle velocity components u_x, u_y, u_z. For such a solution we take τ, Φ, Ψ as new independent variables. Then the desired relationship has the form

$$\tau = \frac{D_\tau}{D} = \frac{(x - x_s)\cos\varepsilon - (y - y_s)\sin\varepsilon + (z_0 - z_s)\operatorname{ctg}\beta}{u_x\cos\varepsilon - u_y\sin\varepsilon + u_z\operatorname{ctg}\beta}, \tag{21}$$

where D and D_τ are the determinants of the homogeneous and inhomogeneous systems (18)-(20). It is of interest that Eq. (21) does not explicitly contain the parameters defining the aerodynamic resistance forces, and coincides with the analogous expression for the case F = 0. However the initial components u_x, u_y, u_z of the velocity which the particle must possess in order to fall upon the vehicle during the time in question do depend on these parameters.

In Eqs. (10), (11), (14)-(16) the unknown time t_0 appears as a parameter. This quantity can be eliminated from these equations if we consider that $t - t_0 = \tau + t_1 - t_0$. Moreover, comparison of Eqs. (8) and (12) for $t = t_1$ gives

$$\sin\omega(t_0 - t_1) = v_{\eta 1}/v_{\perp 1} = (1/v_{\perp 1})(u_x\sin\varepsilon + u_y\cos\varepsilon).$$

Hence, with consideration of Eq. (17) we have

$$\cos\omega(t_0 - t_1) = -(1/v_{\perp 1})[(V_0 + u_z)\sin\beta + v_{\xi 1}\cos\beta].$$

The sign here is chosen from physical considerations, since the phase $\omega(t_0 - t_1)$ is close to π.

Substitution of these expressions in Eq. (10), (11), (14)-(16) leads to the following expression for the particle velocity components relative to the magnetic field (i, j = 1, 2, 3 = x, y, z):

$$v_i = \sum_{j=1}^{3} p_{ij}u_j + \sum_{j=1}^{3} q_{ij}u_j e^{-\delta(t-t_1)} \sin\omega(t-t_1) + \sum_{j=1}^{3} r_{ij}u_j e^{-\delta(t-t_1)} \cos\omega(t-t_1). \tag{22}$$

Here

$$u_1 = u_x;\ u_2 = u_y;\ u_3 = u_z + V_0; \tag{23}$$

$$p_{11} = \cos^2\varepsilon(1+\cos^2\beta);\ p_{12} = -(\sin 2\varepsilon/2)(1+\cos^2\beta);$$

$$p_{13} = \cos\varepsilon\sin 2\beta/2;\ q_{11} = \sin 2\varepsilon\cos\beta;\ q_{12} = \cos 2\varepsilon\cos\beta;$$

$$q_{13} = \sin\varepsilon\sin\beta;\ r_{11} = \sin^2\varepsilon - \cos^2\varepsilon\cos^2\beta;\ r_{12} = (\sin 2\varepsilon/2)\times$$

$$\times(1+\cos^2\beta);\ r_{13} = -\cos\varepsilon\sin 2\beta/2;\ p_{21} = (\sin 2\varepsilon/2)(1+\cos^2\beta);$$

$$p_{22} = \sin^2\varepsilon(1+\cos^2\beta);\ p_{23} = -\sin\varepsilon\sin 2\beta/2;$$

$$q_{21} = \cos 2\varepsilon\cos\beta;\ q_{22} = -\sin 2\varepsilon\cos\beta;\ q_{23} = \cos\varepsilon\sin\beta;$$

$$r_{21} = (\sin 2\varepsilon/2)(1+\cos^2\beta);\ r_{22} = \cos^2\varepsilon - \sin^2\varepsilon\cos^2\beta;$$

$$r_{23} = \sin\varepsilon\sin 2\beta/2;\ p_{31} = -\cos\varepsilon\sin 2\beta/2;\ p_{32} = \sin\varepsilon\sin 2\beta/2;$$

$$p_{33} = \cos^2\beta;\ q_{31} = -\sin\varepsilon\sin\beta;\ q_{32} = -\cos\varepsilon\sin\beta;\ q_{33} = 0;$$

$$r_{31} = \cos\varepsilon\sin 2\beta/2;\ r_{32} = -\sin\varepsilon\sin 2\beta/2;\ r_{33} = \sin^2\beta.$$

Integrating Eq. (22), we obtain an expression for the particle displacements along the coordinate axes over time τ

$$x_i - x_{si} = \frac{1}{\omega\left(1+\frac{\delta^2}{\omega^2}\right)}\left(\sum_{j=1}^{3} q_{ij}u_j + \frac{\delta}{\omega}\sum_{j=1}^{3} r_{ij}u_j\right) + \sum_{j=1}^{3} p_{ij}u_j\tau + \tag{24}$$

$$+\frac{1}{\omega\left(1+\frac{\delta^2}{\omega^2}\right)} e^{-\delta\tau}\left[\left(\sum_{j=1}^{3} r_{ij}u_j - \frac{\delta}{\omega}\sum_{j=1}^{3} q_{ij}u_j\right)\sin\omega\tau - \left(\sum_{j=1}^{3} q_{ij}u_j + \frac{\delta}{\omega}\sum_{j=1}^{3} r_{ij}u_j\right)\cos\omega\tau\right],$$

where $x_1 = x$; $x_2 = y$; $x_3 = z$.

An important quantity when operation of measurement equipment sensitive to charged particles is concerned is the particle energy E_z, related to motion relative to the vehicle along the flight path. The quantity E_z can be considered as a parameter in the equation

$$v_z - V_0 = -\sqrt{2E_z/M}, \tag{25}$$

solution of which will give the mass and initial velocity components required of a particle in order that it enter a sensor located on the vehicle surface in a specified time interval after startup. The sign in Eq. (25) is chosen from the condition of particle incidence on the surface.

In order to solve this problem, Eq. (24) is considered as a system of linear inhomogeneous equations, in which u_j are independent variables and $x_i - x_{si}$ are free terms. The system was solved using Cramer's rule. To simplify the expressions obtained it was considered that in cases of practical interest it is true that $V_0\tau \gg x_i - x_{si}$ (i = 1, 2, 3). The values of u_j thus obtained were then substituted for v_z in Eq. (22), and then in Eq. (25), making use of the relationship $u_x \cos\varepsilon - u_y \sin\varepsilon = -u_z \cot\beta$, which follows from Eq. (21) for $u_i\tau \gg x_i - x_{si}$. After cumbersome calculations this procedure produces the following equation for the total rotation angle φ of the particle trajectory (phase of the motion) in the magnetic field ($\varphi = \omega\tau$):

$$\sqrt{\frac{2E_z}{q\tau BV_0^2}}\sqrt{\varphi}\left[1+\cos 2\beta\left(1-e^{-\delta\tau}\frac{\sin\varphi}{\varphi_*}\right) - \frac{(1-e^{-\delta\tau}\cos\varphi)^2}{\varphi_*\sin\varphi}e^{\delta\tau}\cos 2\beta - \right. \tag{26}$$

$$-\frac{\delta_*^2}{\varphi_*}e^{-\delta\tau}\sin\varphi\cos 2\beta + \frac{2\delta_*}{\sin\varphi}(1-e^{-\delta\tau}\cos\varphi)e^{\delta\tau}\cos^2\beta -$$

$$\left. -\frac{\delta_*^2}{\varphi_*\sin\varphi}(1-e^{-\delta\tau}\cos\varphi)^2 e^{\delta\tau}\cos 2\beta\right] = -\sin^2\beta\left[1-e^{-\delta\tau}\frac{\sin\varphi}{\varphi_*} -\right.$$

$$-\frac{(1-e^{-\delta\tau}\cos\varphi)^2}{\varphi_*\sin\varphi}e^{\delta\tau}-\frac{\delta_*}{\sin\varphi}(1-e^{-\delta\tau}\cos\varphi)^2 e^{\delta\tau}\left(1+\frac{\delta_*}{\varphi_*}\right)+$$
$$+\frac{\delta^*}{\sin\varphi}(1-e^{-\delta\tau}\cos\varphi)e^{\delta\tau}-\delta_*\left(1+\frac{\delta_*}{\varphi_*}\right)e^{-\delta\tau}\sin\varphi\Bigg],$$

where

$$\delta_*=\delta/\omega=\delta\tau/\varphi,\quad \varphi_*=\varphi(1+\delta^2\tau^2/\varphi^2). \tag{27}$$

The quantity $\delta\tau$ in Eq. (26) is a function of φ. In fact, if we consider Eq. (26), the fact that $a = (3/4\pi\rho)^{1/3}M^{1/3}$, where ρ is the density of the particle material, and

$$\varphi=\omega\tau=qB\tau/M, \tag{28}$$

then

$$\delta\tau=amnV_0(3\tau/4\rho)^{2/3}(\pi\varphi/qB)^{1/3}=\delta'\varphi^{1/3}. \tag{29}$$

Equation (26) was solved numerically. To clarify the unique features of this solution, we write Eq. (26) in the form

$$Y=F(\varphi)/(\chi(\varphi)\sqrt{\varphi})=-b/\sin^2\beta,$$

where $F(\varphi)$ and $\chi(\varphi)$ are the expressions in brackets on the right and left sides of Eq. (26) respectively; $b=\sqrt{2E_z/(q\tau BV_0^2)}$. Thus, the solution of Eq. (26) is the point of intersection of the curve $Y(\varphi)$ and the straight line $b/\sin^2\beta$ = const. Estimates performed for the condition of incidence of jet particles on the vehicle surface under the action of the force $q[\mathbf{v}\times\mathbf{B}]$, show that at $u \ll V_0$ the phase φ proves to be significantly less than π. Figure 2 shows the function $|Y(\varphi)|$ for various values of the braking parameter δ' for a sample case in which $\beta = 72.5°$, $\varepsilon = 141°$ (curves 1-7 correspond to δ' = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6). An important feature of this characteristic family is that the curves have a minimum at which $Y_m \neq 0$ if $\delta' \neq 0$, so that the particles fall upon the surface with an energy E_z which exceeds some minimum value. This result is produced by the action of aerodynamic resistance forces. Another peculiarity of the curves is that at $\delta' \neq 0$ Eq. (26) has two roots. This follows directly from the original equation for the equation for the energy $E_z = (M/2)(v_z - V_0)^2$, according to which the defined value of the quantity $E_z > E_m$ can be realized either because of high particle mass or low particle velocity. As is evident from Eq. (28), for the first case high phase is characteristic of the particle motion in the magnetic field, while in the second case low phase is characteristic.

For various values of the parameter δ' Fig. 3a shows the dependence of the dimensionless particle mass $(M'=M(q\tau B)^{-1}=\varphi^{-1})$ on the parameter b, a result of solution of Eq. (26). In Figs. 3a, b and 4 curves 1-4 correspond to δ' = 0.04, 0.16, 0.28, and 0.42. The angles β and ε are the same as in Fig. 2. The parmaeter range chosen corresponds to the neutral atom and molecule concentrations at heights of 200-300 km in the terrestrial atmosphere; for the heavy charged particles exiting the engine a range of E_z/q (which appears in the expression for b) of approximately 1-200 J/C was considered. Here and below the letter f denotes that branch of the curves corresponding to phase values $\varphi > \varphi_{min}$ in Fig. 2. In practice the upper limit of the particle velocity is the reactive jet velocity. In Figs. 3 and 4 this limit is shown by the dashed line I for the case where $u_{max} = 0.25\ V_0$. Values of the dependent variables on the f branch corresponding to b values higher than those on the limiting curve (for example, lower mass values in Fig. 3a) cannot be realized. With change in the quantity u_{max}/V_0 over the range 0.2-0.35 the position of this curve changes insignificantly. In some cases another limitation arises from the requirement that the signs of the

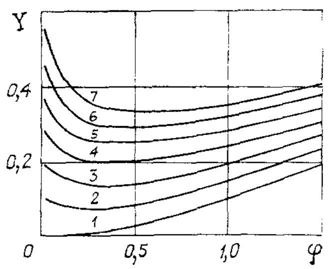

Fig. 2

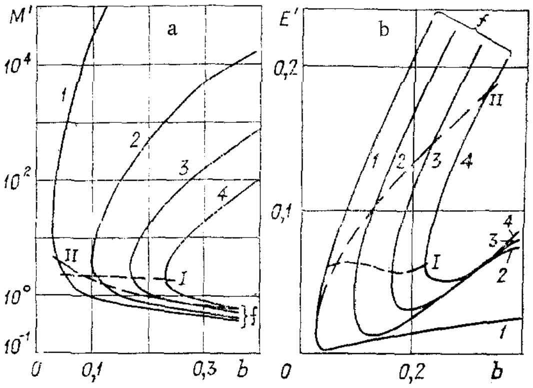

Fig. 3

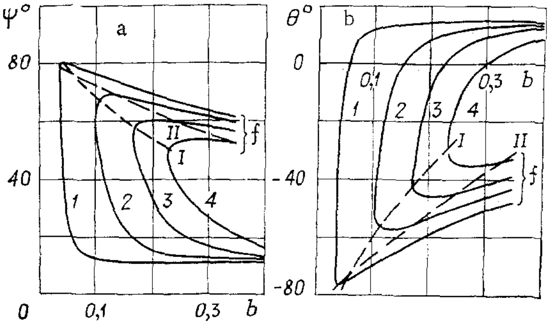

Fig. 4

initial velocity components and the particle coordinates coincide, this following from the gas dynamic conditions of particle escape in the jet. For example, calculations performed for $\delta' = 0$ lead to a contradiction of these conditions. From the solution of the equations at $\delta' = 0$ it follows that the particles must begin their motion in the semispace $z_s > z_0$, having velocity components $u_z < 0$, which does not agree with the gas dynamic conditions. For the case $\delta' \neq 0$ this contradiction can also occur for high phase values φ (in Fig. 3a, the corresponding limitation is shown by the dashed line II).

Recalculation of the quantity M' to a real scale shows that $\tau \simeq 0.2$-1 sec and $Z \sim 1$ ($Z = q/e$, where e is the charge of the electron) and the particle mass M lies in the range of 10^{-20}-10^{-16} g.

The results of calculating total particle energy in a reference frame fixed to the vehicle at the moment of contact between particle and vehicle are shown in Fig. 3b. Calculations were performed with Eqs. (22), (23), with $E' = E(q\tau BV_0^2)^{-1}$. These calculations show that for the conditions considered above the energy E lies in the range from units to hundreds of eV depending on the values of the parameters δ' and b. Figure 4 shows the functions $\psi(b)$ and $\theta(b)$ for angles formed by the particle velocity vector at $q > 0$ and the x and y axes of the vehicle at the finish moment. These angles are defined by

$$\psi = \mathrm{arc\,tg}\,[v_y/(v_z - V_0)],\quad \theta = \mathrm{arc\,tg}\,[v_x/(v_z - V_0)],$$

with $v_z - V_0 < 0$.

An increase in the charge Z which can be carried by the particle with other conditions remaining constant leads to a displacement of the solutions on the branch corresponding to low values of δ' and b. Inasmuch as the parameter δ' depends on Z more weakly than b, growth in Z with E_z = const leads to a b value in the range where there is no solution. It follows from this that with increase in Z the minimum value of E_z must also increase. In addition, calculations show that the total particle energy relative to the vehicle, E, also increases. The required particle mass M also increases.

In addition to calculations of the particle parameters shown in Figs. 3 and 4, using the same conditions calculations were made of the difference in particle start and finish coordinates, as well as the quantities u, u_x, u_y, u_z. These calculations show that for the mutual orientation of the vectors **B** and $\mathbf{V}_0$, shown in Fig. 1, those positively charged particles land on the vehicle surface for which $y - y_s < 0$, $z - z_s < 0$; the difference $x - x_s$ may be either positive or negative depending on the value of the phase φ (i.e., the particle mass). It follows from calculations of the initial velocity projections u_x, u_y, u_z that for such particles $u_z > 0$, aside from the cases which were considered above, and also $u_y > 0$; the quantity u_x may have either sign, depending on the phase φ. It should be noted that solutions for both small and large φ values have a limitation. The parameter values at which this limitation may appear are, for example, minimum distance between particle start position in the jet and finish position on the vehicle surface, defined by the position of the jet boundary, as well as minimum value of the particle initial velocity. Concrete values of these parameters depend to a high degree on the conditions of jet and charged particle formation, and also the coordinates of the point on the vehicle surface which is considered as the finish point.

We will now consider the question of neutralization of the charge on such particles if they are in motion in the ionosphere, i.e., a rarefied plasma. Since the particle size is small ($\sim 10^{-8}$ m) in comparison to the Debye electric field screening radius in the ionosphere ($\sim 10^{-2}$ m) [5], these particles can be considered as Coulomb centers with current charge Z, which is neutralized by capture of particles with opposite sign. Using the expression for particle flux attracted to a Coulomb center (see, for example, [5]), for Z(t) we find

$$Z + (e^2N_bS_p/Ea)Z + N_bS_p = 0. \tag{30}$$

Here N_b is the flux density of attracted particles; E is the attracted particle energy at infinity; S_p is the characteristic collection area. When the attracted particles are electrons, $N_b = (n/4)\langle v_e \rangle$, $E = kT_e$, $S_p = 4\pi a^2$, while n is the electron concentration, $\langle v_e \rangle$ is their mean velocity, and T_e is their temperature (it is considered that $\langle v_e \rangle$ is much greater than the velocity of the Coulomb center). If the original particle is charged negatively, then the attracted particles are ions. The thermal velocity of ions in the ionosphere is low in comparison to the Coulomb center velocity, so that in a coordinate system bound to the ionosphere the center has a velocity of the order of magnitude of V_0 (for $V_0 \gg u$). Therefore for ions $N_b \simeq nV_0$, $E \simeq M_iV_0^2/2$, $S_p = \pi a^2$ (where M_i is the ion mass).

The solution of Eq. (30) has the form

$$Z = Z_0 \exp\left(-\frac{e^2N_bS_p}{Ea}t\right) - \frac{Ea}{e^2}\left[1 - \exp\left(-\frac{e^2N_bS_p}{Ea}t\right)\right].$$

Thus change in charge occurs with a time constant $\tau_n = Ea/(e^2N_bS_p)$. Estimates show that for an electron concentration of $\sim 10^4$ cm^{-3} and electron temperature of ~ 2000°K the value of τ_n is about 3 sec, if the Coulomb center has a radius of $\sim 5 \cdot 10^{-9}$ m. Thus, for sufficiently low electron concentrations, when the condition $\tau \ll \tau_n$ is satisfied, the charge of a positively charged heavy particle can be considered approximately constant. For a negatively charged particle of the same radius this assumption is valid at a concentration of $\sim 10^6$ cm^{-3}. In fact, since $M_iV_0^2/2 \simeq 8$ eV, at $n \sim 10^6$ cm^{-3} and $V_0 \simeq 8$ km/sec we have $\tau_n \simeq 40$ sec, i.e., the condition $\tau \ll \tau_n$ is satisfied.

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